

$$1) \sum_{k=0}^{\infty} \frac{2}{6^{k+1}} = \sum_{k=0}^{\infty} 2 \left(\frac{1}{6}\right)^{k+1}$$

$$S = \frac{\frac{2}{6}}{1 - \frac{1}{6}} = \frac{\frac{1}{3}}{\frac{5}{6}} = \boxed{\frac{2}{5}}$$

$$2) \sum_{k=0}^{\infty} \frac{3}{10^{k+1}} = \sum_{k=0}^{\infty} 3 \left(\frac{1}{10}\right)^{k+1}$$

$$S = \frac{\frac{3}{10}}{1 - \frac{1}{10}} = \boxed{\frac{1}{3}}$$

$$3) a_1 = 1, a_2 = 3, a_n = 2a_{n-1} + 3$$

$$1, 3, 9, 21, 45, 93$$

$$4) a_1 = 1, a_2 = 3, a_n = a_{n-1} + a_{n-2}$$

$$1, 3, 4, 7, 11, 18$$

$$5) 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$$

$$a_n = \frac{1}{2n-1}$$

$$6) 1, -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}$$

$$a_n = (-1)^{n+1} \left(\frac{1}{2n-1}\right)$$

$$7) \sum_{n=1}^{\infty} \frac{1}{(\sqrt{5}-1)^n} = \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{5}-1}\right)^n = S$$

$$|r| = \frac{1}{\sqrt{5}-1} < 1$$

S converges by GST

$$8) \sum_{n=1}^{\infty} \frac{1}{(\sqrt{3}-1)^n} = \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{3}-1}\right)^n = S$$

$$|r| = \frac{1}{\sqrt{3}-1} \geq 1$$

S diverges by GST

$$9) \int_1^{\infty} \frac{e^{x^{-1}}}{x^2} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-2} e^{x^{-1}} dx$$

$$\lim_{b \rightarrow \infty} [-e^{x^{-1}}] \Big|_1^b$$

$$\lim_{b \rightarrow \infty} [-e^{1/b} + e] = \boxed{e-1}$$

10) a) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = S$

$|r| = \frac{1}{2} < 1$

S converges by GST

$S = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$

b) $\sum_{n=1}^{\infty} \frac{1}{n} = S$

$p = 1$

S diverges by p-series test

c) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} = S$

$p = \frac{3}{2} > 1$

S converges by p-series Test

d) $\sum_{n=1}^{\infty} \frac{1}{n^2} = S$

$p = 2 > 1$

S converges by p-series Test

e) $\sum_{n=1}^{\infty} \frac{1}{e^n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n = S$

$|r| = \frac{1}{e} < 1$

S converges by GST

$S = \frac{\frac{1}{e}}{1 - \frac{1}{e}} = \frac{\frac{1}{e}}{\frac{e-1}{e}} = \frac{e-1}{e}$

f) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = S$

$p = \frac{1}{2} < 1$

S diverges by p-series test

g) $\sum_{n=1}^{\infty} n = S$

$\lim_{n \rightarrow \infty} n = \infty \neq 0$

S diverges by n^{th} term Test

h) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n = S$

$|r| = \frac{3}{2} > 1$

S diverges by GST.